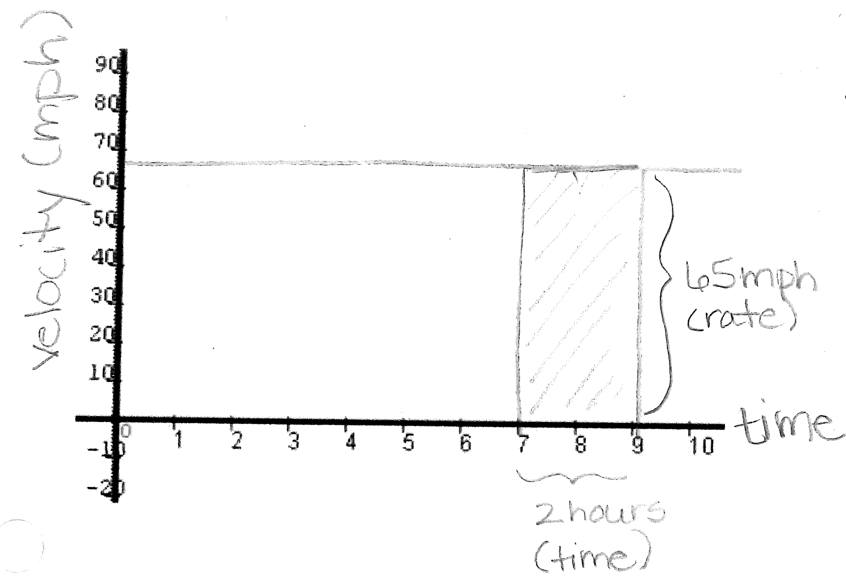
Name Key (2014)

5.1: Estimating Finite Sums

Example: A train moves along a track at a steady rate of 65 mph from 7 am to 9 am. What is the total distance traveled?



$d = r \cdot t$

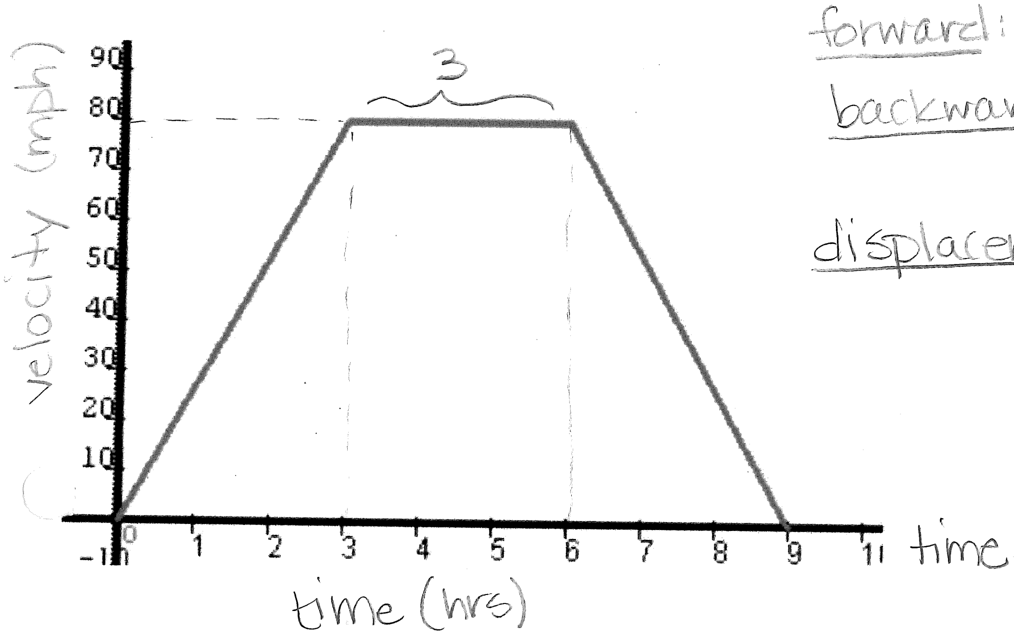
$d = 65\frac{mi}{hr} \cdot 2hr = \boxed{130 \text{ miles}}$

graphical representation: area under velocity curve

Displacement = change in position = Area under velocity curve

Example: Here is the graph of the velocity of a train. Describe the motion of this train.

How far did it travel?



stopped: $t = 0, 9$ hours, b/c $v' = 0$

forward: $(0, 9)$ hours, b/c $v > 0$

backward: never

displacement $= \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(3\text{hrs} + 9\text{ hrs}) \cdot 80\frac{mi}{hr}$

$= \frac{1}{2}(12\cancel{\text{hrs}}) \cdot 80\frac{mi}{\cancel{hr}}$

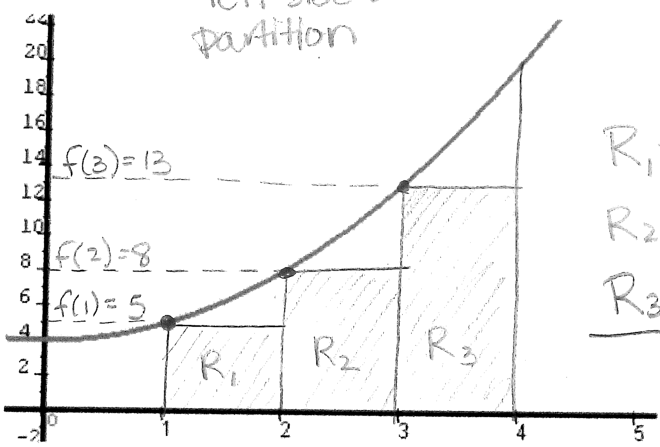
$= \boxed{480 \text{ miles}}$

Trapezoid area $= \frac{1}{2}(b_1 + b_2) \cdot h$

Rectangular Approximation Method (RAM)

Example: Use LRAM to estimate the region bounded by $a = 1$ and $b = 4$ for $f(x) = x^2 + 4$. Use 3 partitions.

height: left side of partition



$R_1 = 1 \cdot f(1) = 1 \cdot 5 = 5$

$R_2 = 1 \cdot f(2) = 1 \cdot 8 = 8$

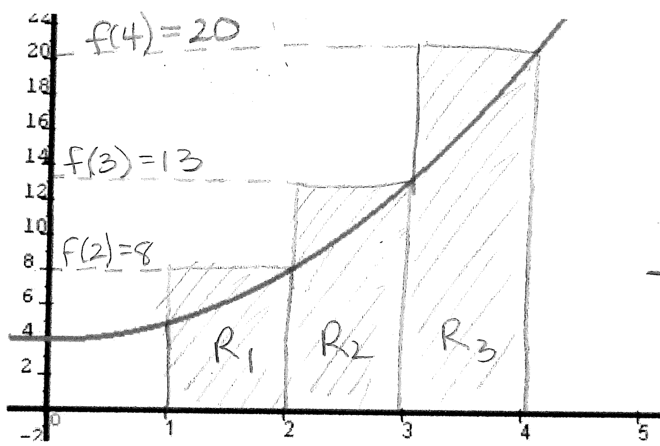
$R_3 = 1 \cdot f(3) = 1 \cdot 13 = 13$

Total area = $26u^2$

estimate is underestimate (b/c f(x) is increasing)

height: right side of partition

Example: Use RRAM to estimate the region.



$R_1 = 1 \cdot f(2) = 8$

$R_2 = 1 \cdot f(3) = 13$

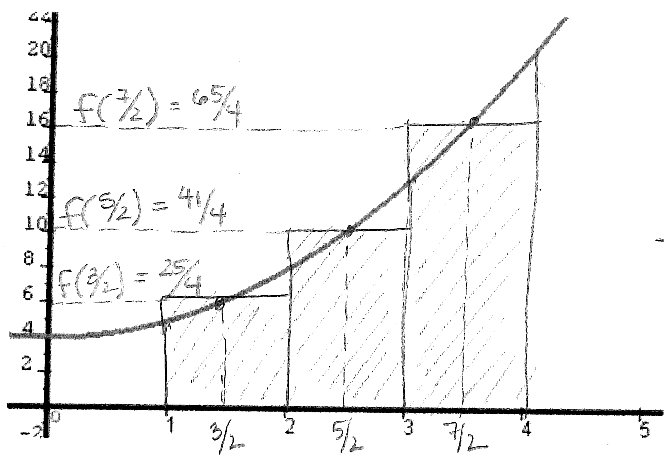
$R_3 = 1 \cdot f(4) = 20$

Total area = $41u^2$

estimate is overestimate (b/c f(x) is decreasing)

height: midpt. of partition

Example: Use MRAM to estimate the region.



$R_1 = 1 \cdot f(3/2) = 1 \cdot \left(\frac{9}{4} + \frac{16}{4}\right) = 25/4$

$R_2 = 1 \cdot f(5/2) = 1 \cdot \left(\frac{25}{4} + \frac{16}{4}\right) = 41/4$

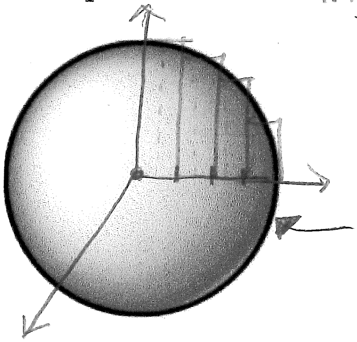
$R_3 = 1 \cdot f(7/2) = 1 \cdot \left(\frac{49}{4} + \frac{16}{4}\right) = 65/4$

Total area = $\frac{131}{4} = 32.75u^2$

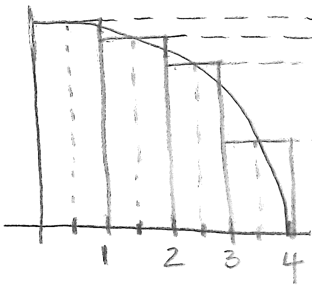
* how to get better estimate? More rectangles!

Example: Estimate the volume of a sphere with cross-section $x^2 + y^2 = 16$.
Use 8 partitions using MRAM

$y = \pm\sqrt{16 - x^2}$



rotate rectangles around x-axis to create cylinders to fill sphere

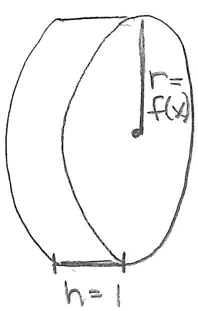


$f(\frac{1}{2}) = \sqrt{16 - (\frac{1}{2})^2} = \sqrt{\frac{63}{4}}$

$f(\frac{3}{2}) = \sqrt{16 - (\frac{3}{2})^2} = \sqrt{\frac{55}{4}}$

$f(\frac{5}{2}) = \sqrt{16 - (\frac{5}{2})^2} = \sqrt{\frac{39}{4}}$

$f(\frac{7}{2}) = \sqrt{16 - (\frac{7}{2})^2} = \sqrt{\frac{15}{4}}$



Cylinder volume $= \pi r^2 h$

$C_1 = \pi\left(\sqrt{\frac{63}{4}}\right)^2(1) = \frac{63\pi}{4}$

$C_2 = \pi\left(\sqrt{\frac{55}{4}}\right)^2(1) = \frac{55\pi}{4}$

$C_3 = \pi\left(\sqrt{\frac{39}{4}}\right)^2(1) = \frac{39\pi}{4}$

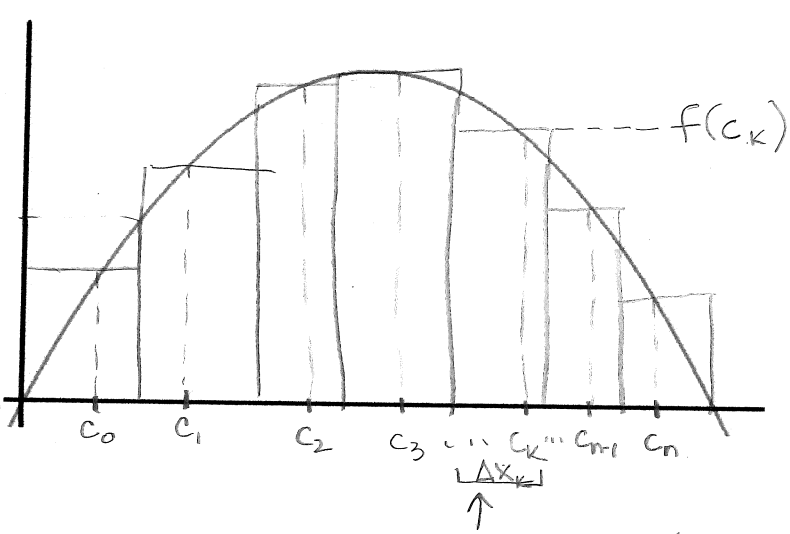
$C_4 = \pi\left(\sqrt{\frac{15}{4}}\right)^2(1) = \frac{15\pi}{4}$

$\frac{1}{2}$ Volume $= \frac{172\pi}{4} = 43\pi$

Total Volume $\approx \boxed{86\pi\ u^3}$

5.2: Definite Integrals

Riemann Sums (like RRAM, LRAM, MRAM) are sums formed by partitioning a closed interval [a, b]. → rectangles used to estimate area



Kth rectangle: Area $= \underbrace{\Delta X_K}_{\text{base}} \cdot \underbrace{f(c_K)}_{\text{height}}$

Sum of n rectangles

$= \sum_{K=1}^{n} f(c_K) \cdot \Delta X_K$

sum of infinite # of rectangles

$= \lim_{n \to \infty} \sum_{K=1}^{n} f(c_K) \cdot \Delta X_K$

Definition:

limits of integration

Definite integral of a continuous function on [a, b].

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\cdot \Delta x_k = \int_a^b f(x)\,dx$$

variable of integration

integral sign: sums infinite # of rect. between a & b

Example: Rewrite $\lim\limits_{n\to\infty} \sum_{k=1}^{n}[4(c_k)^2 - 3(c_k) + 5]\cdot \Delta x$ over $[2,7]$ using calculus notation.

$$\int_2^7 (4x^2 - 3x + 5)\,dx$$

Theorem: Existence of Definite Integrals

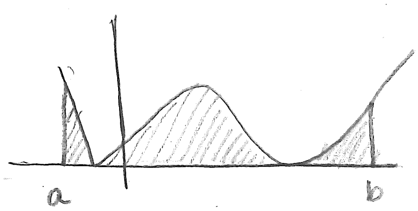
All continuous functions are "integrable" (definite integral exists)

Definition: If $y = f(x)$ is nonnegative and integrable over $[a,b]$, then the area bounded by the curve and the x-axis from a to b is

$f \geq 0$

An area is always positive

$$Area = \int_a^b f(x)dx$$

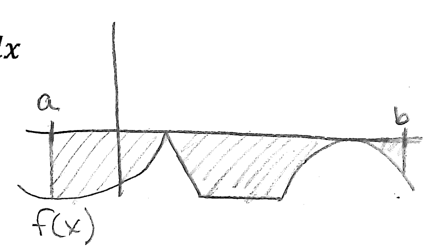


If $y = f(x)$ is nonpositive and integrable over $[a,b]$, then the area bounded by the curve and the x-axis from a to b is

$f \leq 0$

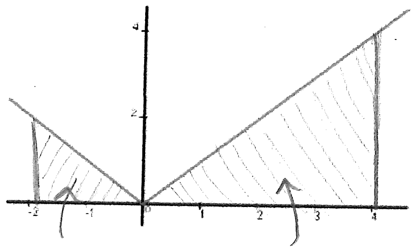
When f(x) below x-axis $\int_a^b f(x)dx < 0$

$$Area = -\int_a^b f(x)dx$$



Example: Evaluate $\int_{-2}^{4}|x|dx$ = positive b/c $f(x)=|x|$ above x-axis

means: find area between $|x|$ and x-axis from $x=-2$ to $x=4$



$A=\frac{1}{2}(2)(2)$
$=2$

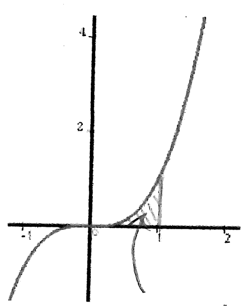
$A=\frac{1}{2}(4)(4)$
$=8$

$\int_{-2}^{4}|x|\cdot dx = 2+8 = \boxed{10}$

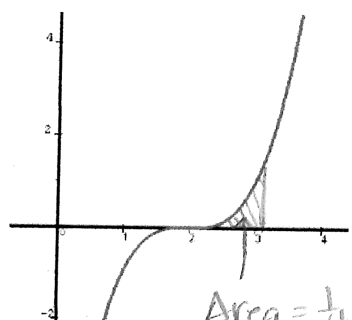
Example: Given $\int_0^1 x^3 dx = \frac{1}{4}$, evaluate $\int_2^3 (x-2)^3 dx$

shift right 2

limits of integration also shifted right 2



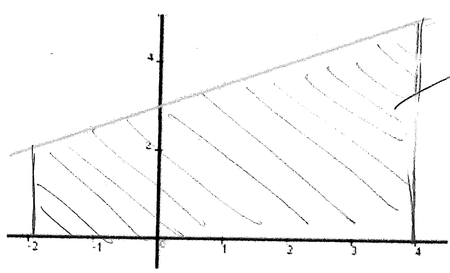
Area $=\frac{1}{4}$



Area $=\frac{1}{4}$

$\int_2^3 (x-2)^3 dx = \boxed{\frac{1}{4}}$

Example: Evaluate $\int_{-2}^{4}\left(\frac{1}{2}x+3\right)dx$



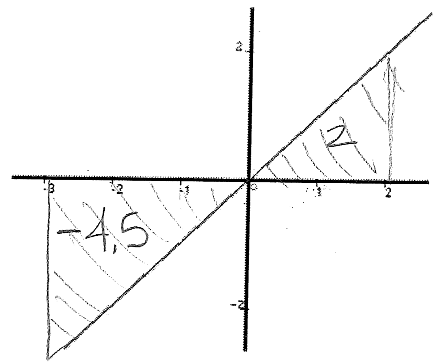
Trapezoid Area:
$A=\frac{1}{2}(b_1+b_2)\cdot h$

$A=\frac{1}{2}(5+2)(6)=21$

$\int_{-2}^{4}\left(\frac{1}{2}x+3\right)dx = \boxed{21}$

Example: Evaluate $\int_0^2 x\,dx$

$= \frac{1}{2}(2)(2) = \boxed{2}$



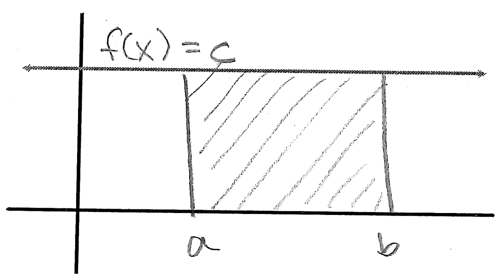
Evaluate $\int_{-3}^0 x\,dx$

$= -\frac{1}{2}(3)(3) = \boxed{-4.5}$

Evaluate $\int_{-3}^2 x\,dx$

$= 2 - 4.5 = \boxed{-2.5}$

$\int_a^b f(x)dx =$ "net area" = area above x-axis − area below

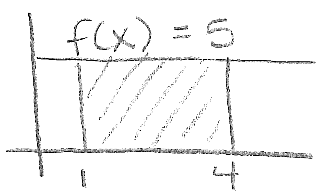


Theorem: The integral of a constant

If f(x) = c where c is constant on [a, b], then

$$\int_a^b f(x)dx = \int_a^b c\,dx = \underbrace{c(b-a)}_{\text{area rectangle}}$$

Example: Evaluate $\int_1^4 5dx$ $= 5(4-1) = 5(3) = \boxed{15}$

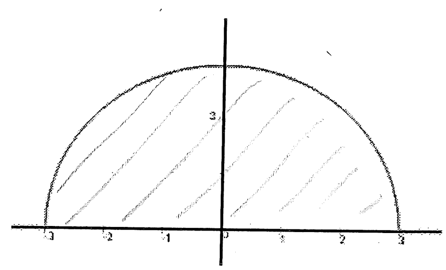


Example: Evaluate $\int_{-3}^3 \sqrt{9-x^2}dx$ $= \frac{1}{2}\pi(3^2)$

$= \boxed{\frac{9\pi}{2}}$

$x^2 + y^2 = r^2$
circle w/
center (0,0)
and radius r

$\frac{1}{2}$ circle
c: (0,0)
r = 3



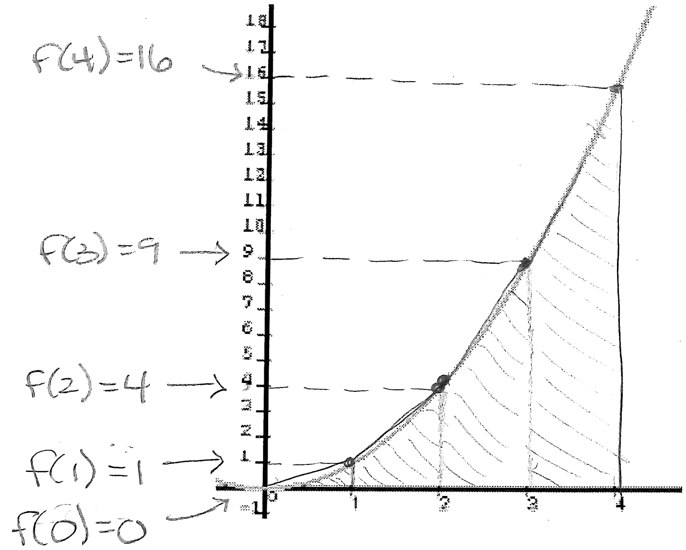
** Definite integrals on TI84: $\int_a^b f(x)dx =$ **fnInt**$(f(x), x, a, b)$

5.5: Trapezoids and Simpson's Rule

Example: Approximate $\int_0^4 x^2 dx$ using trapezoids, n = 4.

$f(x) = x^2$

Trapezoid area $= \frac{1}{2}(b_1 + b_2) \cdot h$



$T_1 = \frac{1}{2}(0+1) \cdot 1 = \frac{1}{2}$

$T_2 = \frac{1}{2}(1+4) \cdot 1 = \frac{5}{2}$

$T_3 = \frac{1}{2}(4+9) \cdot 1 = \frac{13}{2}$

$T_3 = \frac{1}{2}(9+16) \cdot 1 = \frac{25}{2}$

Total $= \frac{44}{2}$

$\boxed{\int_0^4 x^2 dx \approx 22}$

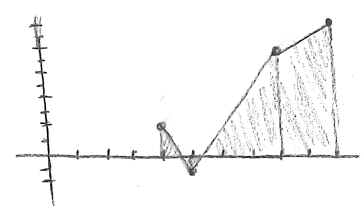
w/ calculator: $64/3 = 21\,1/3$

Example: Use 3 trapezoids to approximate $\int_4^{10} f(x)dx$ given the following information.

heights: 1, 3, 2

x	4	5	8	10
f(x)	2	-1	7	9

← base lengths



*still use trapezoid formulas even though looks like this

$T_1 = \frac{1}{2}(2+-1) \cdot 1 = 1/2$

$T_2 = \frac{1}{2}(-1+7) \cdot 3 = 9$

$T_3 = \frac{1}{2}(7+9) \cdot 2 = 16$

Total $= 25\,1/2$

$\boxed{\int_4^{10} f(x)dx \approx 25\,1/2}$

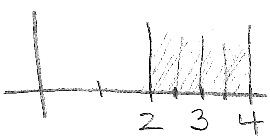
Simpson's Rule:

[a, b] is partitioned into equal subintervals and h = (b - a)/n to approximate $\int_a^b f(x)dx$

$$\boxed{S=\frac{h}{3}\left(y_0+4y_1+2y_2+4y_3+\cdots+2y_{n-2}+4y_{n-1}+y_n\right)}$$

1st y value   2nd y value   last y value

Example: Approximate $\int_2^4 5x^4dx$ using Simpson's Rule with n = 4.



n=4

x	f(x)
2	$5(2)^4=y_0$
5/2	$5(5/2)^4=y_1$
3	$5(3)^4=y_2$
7/2	$5(7/2)^4=y_3$
4	$5(4)^4=y_4$

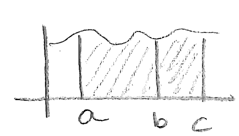
$$S=\frac{h}{3}\cdot 5\left(2^4+4\left(\frac{5}{2}\right)^4+2(3)^4+4\left(\frac{7}{2}\right)^4+4^4\right)$$

$$\approx 992.083$$

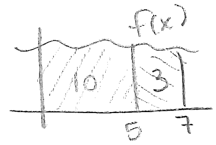
$$\boxed{\int_2^4 5x^4dx\approx 992.083}$$

5.3: Definite Integrals and Antiderivatives

Rules for definite integrals:

Zero	$\int_a^a f(x)dx=0$
Constant Multiple	$\int_a^b k\cdot f(x)dx=K\cdot\int_a^b f(x)dx$
Order of Integration	$\int_a^b f(x)dx=-\int_b^a f(x)dx$
Sum and Difference	$\int_a^b[f(x)\pm g(x)]dx=\int_a^b f(x)dx\pm\int_a^b g(x)dx$
Additivity	$\int_a^b f(x)dx+\int_b^c f(x)dx=\int_a^c f(x)dx$ 

Example: Given $\int_0^5 f(x)dx = 10$ and $\int_5^7 f(x)dx = 3$, find:



$\int_0^5 3f(x)dx =$

$3\int_0^5 f(x)dx = 3(10) = \boxed{30}$

$\int_5^5 f(x)dx = 0$

$\int_5^0 f(x)dx = -\int_0^5 f(x)dx$

$= \boxed{-10}$

$\int_0^7 f(x)dx =$

$= (10+3)$

$= \boxed{13}$

Example: Given $\int_2^6 f(x)dx = 12$ and $\int_2^6 g(x)dx = -2$, find:

$\int_2^6 [f(x) + g(x)]dx =$

$\int_2^6 f(x)dx + \int_2^6 g(x)dx$

$= 12 + -2 = \boxed{10}$

$\int_2^6 2g(x)dx =$

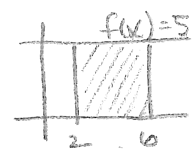
$2\int_2^6 g(x)dx = 2(-2) = \boxed{-4}$

$\int_2^6 [g(x) - f(x)]dx =$

$\int_2^6 g(x)dx - \int_2^6 f(x)dx$

$= -2 - 12 = \boxed{-14}$

$\int_2^6 5dx =$



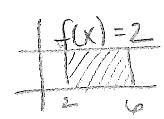
$5(6-2) = 5 \cdot 4 = \boxed{20}$

$\int_2^6 [f(x) + 5]dx =$

$\int_2^6 f(x)dx + \int_2^6 5dx$

$= 12 + 20 = \boxed{32}$

$\int_2^6 [2 - g(x)]dx =$



$\int_2^6 2dx - \int_2^6 g(x)dx$

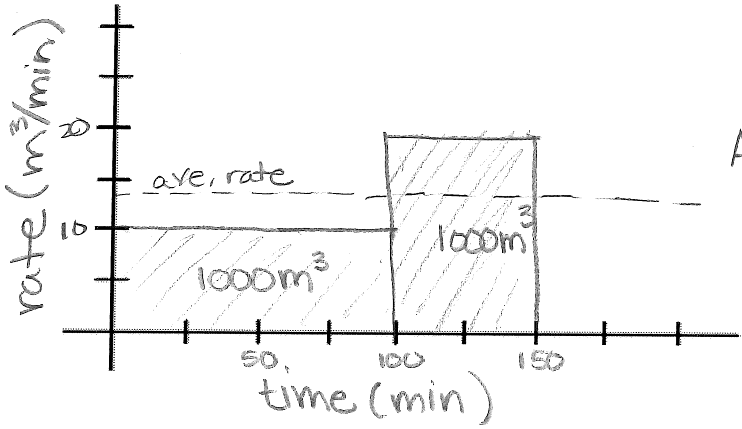
$= 2(6-2) - -2$

$= 2(4)+2 = 8+2 = \boxed{10}$

how long would this take?

Example: A dam released 1000 m³ of water at 10 m³/min and then released another 1000 m³ at 20 m³/min . What was the average rate at which the water was released?

answer is not 15 m³/min... why?



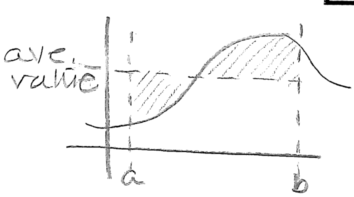
$$\text{Ave rate} = \frac{(1000+1000)\,m^3}{150\,min} = 13\tfrac{1}{3}\ m^3/min$$

The ***average value*** of a function on [a, b]:

$$\boxed{\text{Ave value of } f = \frac{\int_a^b f(x)\,dx}{b-a}}$$

net area under curve

width of interval



ave value = height of "sand" smashed into rectangle

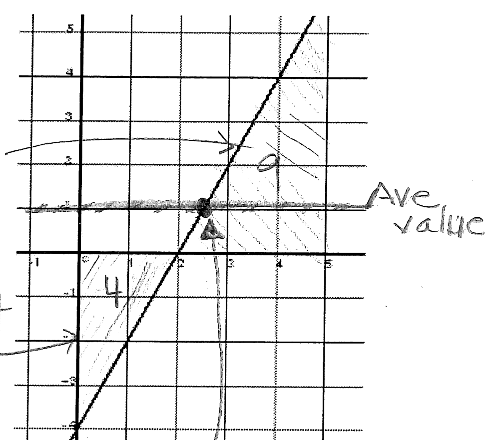
Example: Find the average value of $f(x) = 2x - 4$ on $[0, 5]$

$$\text{Ave value} = \frac{\int_0^5 (2x-4)\,dx}{5-0}$$

$$= \frac{9-4}{5} = \frac{5}{5} = 1$$

$A = \frac{1}{2}(3)(6) = 9$

$A = \frac{1}{2}(2)(4) = 4$



- Where does the function *equal* the average value on the interval [0,5]?

$$f(x) = \frac{\int_0^5 f(x)\,dx}{5-0}$$

$$2x - 4 = 1$$

$$2x = 5$$

$$x = 5/2$$

Mean Value Theorem for Definite Integrals:

If f is continuous on [a,b], then at some point c in [a,b]

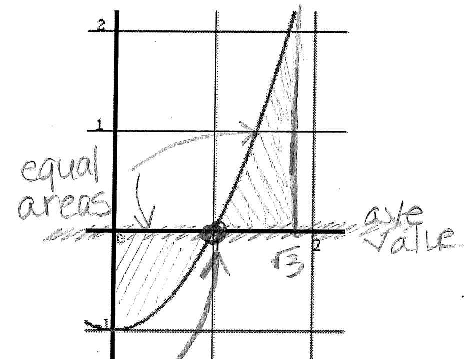
$$f(c) = \frac{\int_a^b f(x)\,dx}{b-a}$$

Example: Find the c that is guaranteed by the MVT for integrals for $y = x^2 - 1$ on $[0, \sqrt{3}]$.

step 1: Find the average value

$$\text{ave. val.} = \frac{\int_0^{\sqrt{3}} (x^2 - 1)\,dx}{\sqrt{3} - 0}$$ use calculator (math 9)

$$= \boxed{0}$$

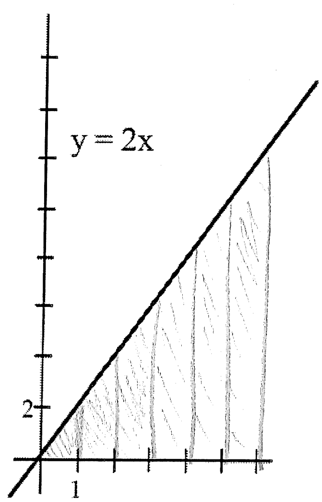


step 2: Find point(s) where function = average value

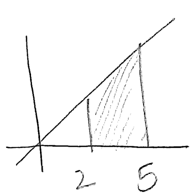
f(x) = ave. value

$x^2 - 1 = 0$

$x = \pm 1$ → $\boxed{x = 1 \text{ on } [0, \sqrt{3}]}$



b	$\int_0^b 2x\,dx$
1	$\int_0^1 2x\,dx = \frac{1}{2}(1)(2) = 1$
2	$\int_0^2 2x\,dx = \frac{1}{2}(2)(4) = 4$
3	$\int_0^3 2x\,dx = \frac{1}{2}(2)(9) = 9$
4	$\int_0^4 2x\,dx = \frac{1}{2}(2)(16) = 16$
5	$\int_0^5 2x\,dx = \frac{1}{2}(2)(25) = 25$
6	$\int_0^6 2x\,dx = \frac{1}{2}(2)(36) = 36$



$\int_2^5 2x\,dx = 25 - 4 = 19$

x^2@5 x^2@2

Fundamental Theorem of Calculus part II

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

net area under curve

antiderivative of f

$\frac{d}{dx}(F) = f$

Examples: Evaluate. using calculus… finally! ☺

$\int_2^5 (3x^2+2x)dx$

$= x^3+x^2+c \Big|_2^5$

$= 5^3+5^2+c-(2^3+2^2+c)$

$= 125+25+c-(8+4+c)$

$= 150+c-12-c = 138$

"c" will always cancel for definite integrals

$\int_0^{3\pi/4} \sec x \tan x\, dx$

$= \sec x \Big|_0^{3\pi/4}$

$= \sec\frac{3\pi}{4} - \sec 0$

$= \boxed{-\frac{2}{\sqrt{2}} - 1}$

Evaluate $\int_{-1}^{1} \frac{1}{1+x^2} dx$

$= \tan^{-1} x \Big|_{-1}^{1}$

$= \tan^{-1} 1 - \tan^{-1}(-1)$

$= \pi/4 - -\frac{\pi}{4}$

$= \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$

Evaluate $\int_0^1 e^x dx$

$= e^x \Big|_0^1$

$= e^1 - e^0$

$= \boxed{e-1}$

Example: If $s(t) = 3t+5$ (feet) is the position of a particle at time t (seconds), find the displacement on [0,4].

disp $= s(4)-s(0)$

$= 3(4)+5-[3(0)+5]$

$= 12+5-[0+5]$

$= \boxed{12 \text{ ft}}$ "net" distance traveled

not a coincidence

Example: If $v(t) = 3$ ft/sec is the velocity of a particle at t, find the definite integral of the velocity from $t = 0$ to $t = 4$.

$v(t) = s'(t) = 3$

$\int_0^4 v(t)dt = \int_0^4 3\, dt$

$= 3t \Big|_0^4 = 3\cdot 4 - 3\cdot 0 = \boxed{12 \text{ ft}}$

displacement $= \int_a^b v(t)\,dt = s(b)-s(a)$

5.4: Fundamental Theorem of Calculus

$\int_a^b f(x)dx = F(b) - F(a)$

Example: Evaluate using the Fundamental Theorem of Calculus.

$\int_{-1}^{3}(x^2 + 3x)dx$

$$= \frac{x^3}{3} + \frac{3x^2}{2}\Big|_{-1}^{3} = \frac{3^3}{3} + \frac{3}{2}\cdot 3^2 - \left[\frac{(-1)^3}{3} + \frac{3(-1)^2}{2}\right]$$

$$= \frac{27}{3} + \frac{27}{2} - \frac{-1}{3} - \frac{3}{2} = \frac{28}{3} + \frac{24}{2}$$

$$= 9\tfrac{1}{3} + 12 = \boxed{21\tfrac{1}{3}}$$ "net area"

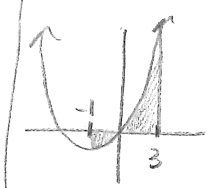
On calculator:

graph: $Y_1 = x^2 + 3x$

calc: $\int f(x)dx$

lower: $x = -1$

upper: $x = 3$



Calculate the *total* area between the curve y = x² +3x and the x-axis on [-1, 3].

positive

below x-axis

above x-axis

changes to +

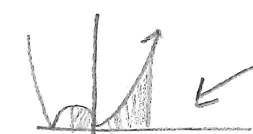
$$-\int_{-1}^{0} f(x)dx + \int_{0}^{3} f(x)dx = -\left[\frac{0^3}{3} + \frac{3\cdot 0^2}{2} - \left(\frac{(-1)^3}{3} + \frac{3(-1)^2}{2}\right)\right] + \left[\frac{3^3}{3} + \frac{3\cdot 3^2}{2} - \left(\frac{0^3}{3} + \frac{3\cdot 0^2}{2}\right)\right]$$

$$= -\left[-\left(\frac{1}{3} + \frac{1}{2}\right)\right] + \frac{27}{3} + \frac{27}{2}$$

$$= \frac{5}{6} + \frac{54}{6} + \frac{81}{6} = \frac{140}{6} = \boxed{23\tfrac{1}{3}}$$

"Total area"

On calculator:



$\int_{-1}^{3} |f(x)|dx = 23.\overline{3}$

Example: If $v(t) = 4t^2 - 3t - 1$ (m/sec), what is the displacement between $t = 0$ and $t = 2$?

$$\boxed{\text{disp} = \int_a^b v(t)dt}$$

$$\text{disp} = \int_0^2 (4t^2 - 3t - 1)dt = 4\frac{t^3}{3} - 3\cdot\frac{t^2}{2} - t\Big|_0^2 = \frac{4}{3}\cdot 2^3 - \frac{3}{2}\cdot 2^2 - 2 - 0$$

$$= \frac{32}{3} - \frac{12}{2} - 2 = 10\tfrac{2}{3} - 6 - 2 = \boxed{2\tfrac{2}{3}\text{ meters}}$$

Calculate the *total* distance traveled on [0, 2].

$$\boxed{\text{dist} = \int |v(t)|dt}$$

w/out calc, separate forward & backwards motion

$V = 4t^2 - 3t - 1$

$V = (4t+1)(t-1) = 0$

$t = \cancel{-1/4}, 1$

	0	1
$(4t+1)$	+	+
$(t-1)$	−	+
V	−	+

backwards forwards

to change +

$$\text{dist} = -\int_0^1 v(t)dt + \int_1^2 v(t)dt$$

$$= -\left(\frac{4}{3}\cdot 1^3 - \frac{3}{2}\cdot 1^2 - 1 - 0\right) + \frac{4}{3}\cdot 2^3 - \frac{3}{2}\cdot 2^2 - 2 - \left(\frac{4}{3}\cdot 1^3 - \frac{3}{2}\cdot 1^2 - 1\right)$$

$$= -\left(\frac{8}{6} - \frac{9}{6} - \frac{6}{6}\right) + \frac{64}{6} - \frac{36}{6} - \frac{12}{6} - \left(\frac{8}{6} - \frac{9}{6} - \frac{6}{6}\right)$$

$$= \frac{7}{6} + \frac{16}{6} + \frac{7}{6} = \frac{30}{6} = \boxed{5\text{ meters}}$$

check w/ calculator

$\int_0^2 |v(t)|dt = 5$

Example: Evaluate $\frac{d}{dx}\int_4^x t^2\,dt = \frac{d}{dx}\left(\frac{t^3}{3}\Big|_4^x\right) = \frac{d}{dx}\left(\frac{x^3}{3} - \frac{4^3}{3}\right) = \frac{3x^2}{3} - 0 = x^2$

just changed to x
(not a coincidence!)

Fundamental Thm Part 1:
If f is continuous on [a,b] and $F(x) = \int_a^x f(t)dt$ then,

$F'(x) = \underbrace{\frac{d}{dx}\int_a^x f(t)dt}_{\text{derivative of antiderivative}} = f(x)$ a = any constant

function in terms of x

Examples: Evaluate.

$\frac{d}{dx}\int_{1/2}^x \sin t\,dt = \sin x$

$\frac{d}{dx}\int_5^x (\ln(\csc^{-1} t) + e^{\tan t - 1})dt$

$= \ln(\csc^{-1}x + e^{\tan x - 1})$

$\frac{d}{dx}\int_x^3 \cos t\,dt = \frac{d}{dx}\left(-\int_3^x \cos t\,dt\right)$

$= -\cos x$

constant must be on bottom

Examples: Find $\frac{dy}{dx}$.

not just x... now what??

$y = \int_3^{2x}(2t^2+1)dt$

LONG WAY

$\frac{dy}{dx} = \frac{d}{dx}\int_3^{2x}(2t^2+1)dt = \frac{d}{dx}\left(\frac{2t^3}{3} + t\right)\Big|_3^{2x}$

$= \frac{d}{dx}\left[2\cdot\frac{(2x)^3}{3} + (2x) - \left(2\cdot\frac{(3)^3}{3} + 3\right)\right]$

$= 2\cdot\frac{3(2x)^2\cdot 2}{3} + 2 - 0 = 4(2x)^2 + 2$

$= \boxed{2(2(2x)^2+1)}$

derivative of 2x

sub in 2x

sub in x^2 and mult. by its derivative

$y = \int_1^{x^2}\cos t\,dt$

SHORT CUT

F.T.C ☺

$= \cos(x^2)\cdot 2x$

$= \boxed{2x\cos(x^2)}$

$y = \int_{-4x}^{3x^2-x}\frac{1}{2+e^t}dt$

must be constant

$= \int_{-4x}^{0}\frac{1}{2+e^t}dt + \int_0^{3x^2-x}\frac{1}{2+e^t}dt = -\int_0^{-4x}\frac{1}{2+e^t}dt + \int_0^{3x^2-x}\frac{1}{2+e^t}dt$

same (any constant)

$= \frac{-1}{2+e^{-4x}}\cdot(-4) + \frac{1}{2+e^{3x^2-x}}\cdot(6x-1)$

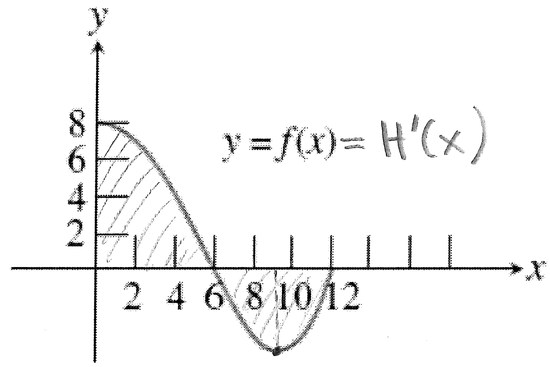
$= \boxed{\frac{4}{2+e^{-4x}} + \frac{6x-1}{2+e^{3x^2-x}}}$

Example: $H(x) = \int_0^x f(t)dt$ where f is the continuous function with domain $[1, 12]$.

$H'(x) = \frac{d}{dx}\int_0^x f(t)dt = f(x)$

a) Find $H(0)$.

$H(0) = \int_0^0 f(t)dt$
$= \boxed{0}$



b) On what interval(s) is H increasing?

$\boxed{[0, 6)}$ when $H' > 0$

c) On what interval(s) is the graph of H concave up?

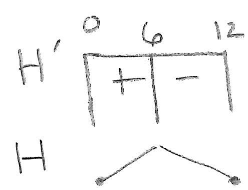
$\boxed{(9, 12]}$ when $H'' > 0$
(slope of $H' > 0$)

d) Is $H(12)$ positive or negative?

$H(12) = \int_0^{12} f(t)dt$ = net area from 0 to 12 > 0 $\boxed{\text{positive}}$
(more area above x-axis than below)

e) Where does H achieve its maximum value?

when H' changes from + to −, or endpts
@ $\boxed{x = 6}$



f) Where does H achieve its absolute minimum value?

When H' changes from − to +, or at endpts

$x = 0$ or $x = 12$ ... Where is abs. min?

$H(0) = 0$ (see part a)
$H(12) > 0$ (see part d)

min value occurs @ $\boxed{x = 0}$